STOCHASTIC IONIZATION OF RELATIVISTIC HYDROGEN-LIKE ATOM

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Abstract

Stochastic ionization of higly excited relativistic hydrogenlike atom in the monochromatic field is considered. A theoretical analisis of chaotic dynamics of the electron based on Chirikov criterion is given. Critical value of the external field is evaluated analitically.

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Introduction

Study of behaviour of higly exited atoms in microwave field is important problem which lies at the intersection of several lines of contemporary research. On of important of this themes is chaos. Recently chaotic dynamics of atom in the microwave field have been subject of extensive reseach. Much theoretical and experimental work was devoted to this problem [1, 2, 3]. A theoretical analisis of behaviour of classical hydrogen atom, based on Chirikov criterion [3, 4] shows that for some critical value of field strength, the electron enters a chaotic regime of motion, marked by unlimited diffusion leading to ionization. Up to now much of the discussion on resonance overlap and chaos has largely been limited by nonrelativistic systems. Investigation of classical relativistic systems with chaotic dynamics has became of subject of extensive research in the last years [11]-[14]. Among these considered problems are some nonlinear oscillators[11] relativistic two-center Kepler problem[12], etc. In our opinion, the study of chaotic dynamics of relativistic atoms interacting with microwave field makes possible to investigate the new phenomenon, namely, relativistic quantum chaos.

In the present paper we generalize mentioned above results on chaotic ionization of classical nonrelativistic hydrogen atom to the case of classical relativistic hydrogen-like atom. For simplicity we consider the one-dimensional model which allows to estimate analitically the values of critical field strength. As is well known (see [8]) and references therein, in the nonrelativistic case one of the remarkable futures of the comparison of the classical theory with the experimental measurements of microwave ionization is the fact that a one-dimensional model of a hydrogen atom in an oscillating electric field provides an excellent description of experimental ionization thresholds for real three-dimensional hydrogen atom. In the case of relativistic hydrogen-like atom one-dimensional

model allows to avoid the difficulties connected with the non-closeness of trajectories in the relativistic Kepler motion [15] and arising additional degrees of freedom [16].

Using Chirikov's criterion of stochasticity we obtain, in terms of action and charge, the analitical formula for the critical value of microwave field, at which stochastic ionization will occur. In this paper we will use the system of units $m_e = h = c = 1$ in which $e^2 = 137^{-1}$.

Consider classical relativistic electron in a field of one-dimensional $-\frac{Z\alpha}{x}$ potential where Z is the charge of the center, $\alpha = \frac{1}{137}$. Momentum of this electron is given by

$$p = \sqrt{(\varepsilon + \frac{Z\alpha}{x})^2 - 1},$$

where ε is the full energy of the electron. Action is defined as

$$n = \int_{x_2}^{x_1} pdx = \pi a\sqrt{1-\varepsilon^2},$$

where $a = \frac{\varepsilon Z\alpha}{1-\varepsilon^2}$ x_1 and x_2 are the turning points of the electron. From this expression we have for unperturbed Hamiltonian

$$H_0 = \frac{n}{\sqrt{n^2 + \pi^2 Z^2\alpha^2}} \tag{1}$$

The corresponding frequency is

$$\omega_0 = \frac{dH_0}{dn} = \frac{\pi^2 Z^2\alpha^2}{(n^2 + \pi^2 Z^2\alpha^2)^{\frac{3}{2}}} \tag{2}$$

It is easy to see that in the nonrelativistic limit (i.e., for small Z) both Hamiltonin and frequency coincide with corresponding nonrelativistic ones [1, 5]. Now we consider interaction of our atom with monchromatic field which is written in the form

$$V(x,t) = \epsilon x cos\omega t, \tag{3}$$

where ϵ and ω are the field amplitude and frequency. First we need to write (3) in the action-angle variables. This can be done by expanding perturbation

(3) into Fourier series:

$$V(x,t) = \epsilon \sum_{-\infty}^{\infty} x_k(n) cos(k\theta - \omega t), \tag{4}$$

where Fourier amplitudes of the perturbation are defined by the integral

$$x_k = \int_0^{2\pi} d\theta e^{im\theta} x(\theta, n) = -\frac{a}{k} J'(ek) = -\frac{n\sqrt{n^2+Z^2}}{k} J'(ek), \tag{5}$$

J'_k is the ordinary Bessel function of order k, $e = \frac{\sqrt{n^2+Z^2}}{n}$.

Thus full Hamiltonian of the relativistic hydrogenlike atom in the monochromotic field can be written in the form

$$H = \frac{n}{\sqrt{n^2+\pi^2 Z^2\alpha^2}} + \epsilon \sum_{-\infty}^{\infty} x_k(n) cos(k\theta - \omega t). \tag{6}$$

For sufficiently small electric fields the Kolmogorov-Arnol'd-Moser theorem [10] guarantes that most of the straight-line trajectories in action-angle space will be only slightly distorted by the perturbation. The maximum distortion of the orbits will occur at resonances where the phase, $k\theta - \omega t$, is stationary [10]. The resonance frequency and actions are threrefore detaermined by the relation

$$k\omega_0 - \omega = 0 \tag{7}$$

Then using Eqs. (2) and (7) the action resonant with the kth subharmonic of the perturbation is

$$n_k = [(\frac{\pi^2 k Z^2\alpha^2}{\omega^2})^{\frac{2}{3}} - \pi^2 Z^2\alpha^2]^{\frac{1}{2}} \tag{8}$$

As is well known there are several methods for investigating of chaotical dynamicsof the system with Hamiltonian in the form (6). Simplest of them is one, developed by Zaslavsky and Chirikov [4, 6, 7] which is called Chirikov criterion. According to this criterion chaotical motion occurs when two neighbouring resonances overlap i.e., when the following condition is obeyed :

$$\frac{\Delta n_k}{\delta n_k} > 1, \tag{9}$$

where Δn_k is the width of k th resonace, $\delta n_k = n_{k+1} - n_k$ is the separation of the k and $k+1$ resonances. Using (8) this separation is

$$\delta n_k = (\frac{kZ^2}{\omega})^{\frac{2}{3}} \frac{1}{3kn_k}$$

According to [7, 10] the width of kth resonance defined as

$$\Delta n_k = 4(\frac{\epsilon x_k}{\omega_0'})^{\frac{1}{2}} \tag{10}$$

where

$$\omega_0' = \frac{d\omega_0}{dn} = \frac{3n_k Z^2}{(n_k^2 + Z^2)^{\frac{5}{2}}}.$$

Note that these formulas for Δn_k and δn_k in the limit of small Z coincides for known formulas corresponding to nonrelativistic case [1].

Taking into account expression for a and using the asymptotic formula $J_k'(ek) \approx 0.411k^{-\frac{5}{3}}$ (for $k >> 1$) for the resonance width we obtain

$$\Delta n_k \approx [\epsilon k^{-\frac{8}{3}} Z^{-3} (n_k^2 + \pi^2 Z^2 \alpha^2]^{\frac{1}{2}}$$

Inserting expressions for Δn_k and δn_k into (9) we have

$$\epsilon^{\frac{1}{2}} k^{-\frac{1}{3}} Z^{-\frac{3}{2}} n_k (n_k^2 + \pi^2 Z^2 \alpha^2)^{\frac{1}{2}} > 1$$

This gives us the critical value of the field amplitude at which sotchastic ionization of the relativistic electron binding in the field of charge $Z\alpha$ will occur:

$$\epsilon_{cr} = k^{\frac{2}{3}} (\pi Z\alpha)^3 n_k^{-2} (n_k^2 + \pi^2 Z^2 \alpha^2)^{-1}$$

For small Z last formula can be expanded into series as follow:

$$\epsilon_{cr} = k^{\frac{2}{3}} Z^3 n_k^{-4} (1 - \frac{Z^2}{n_k^2} + ...)$$

or

$$\epsilon_{cr} = \epsilon_{nonrel} (1 - \frac{Z^2}{n_k^2} + ...)$$

where ϵ_{nonrel} is the critical field corresponding to the nonrelativistic case.

As seen from this formula the critical field requiring for stochastic ionization of relativistic hydrogen-like atom is less than the corresponding nonrelativistic one.

We have obtained approximate analitical formula for the critical value of the field requiring for stochastic ionization of relativistic electron binding in the Coulomb field of charge Z in terms of Z and action n_k. Since relativistic Rydberg atom is an esentially quantum object, the study of its microwave field excitation, provides, therefore an ideal testing ground for the existence of quantum relativictic "chaotic" phenomena. More detail analisis of considered above problem should be given by solving the time-dependent Dirac equation and classical equations of motion. This will be subject of our future publications.

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